INTERNATIONAL BACCALAUREATE

Mathematics: applications and interpretation

MAI

EXERCISES [MAI 2.11-2.12]

EXPONENTS AND LOGARITHMS - II

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A. Paper 1 questions (SHORT)

LOGARITHMS

1. [Maximum mark: 12] ***[try to solve without GDC]***

Write down the following values

$\log_2 16 =$
$\log_3 9 =$
$\log_3 27 =$
$\log_5 1 =$
$\log_5 5 =$
$\log_5 25 =$
$\log 1 =$
$\log 10 =$
$\log 10000 =$
$\log_3 \frac{1}{3} =$
$\log_3 \sqrt{3} =$
$\log 0.1 =$

2. [Maximum mark: 8] ***[try to solve without GDC]***

Find the value of x in the following equations

(a) (i) $\log_2 x = 3$ (ii) $\log_x 8 = 3$ [2]

(b) (i) $\log_5 x = 2$ (ii) $\log_x 25 = 2$ [2]

(c) (i) $\log_4 2x = 2$ (ii) $\log_{2x} 16 = 2$ [2]

(d) (i) $\log x = 2$ (ii) $\ln x = 4$ [2]

3. [Maximum mark: 6]

Find the following values in terms of k

$\log 10^k =$	$\ln e^k =$
$\log \frac{1}{10^k} =$	$\ln \frac{1}{e^k} =$
$\log \sqrt[k]{10} =$	$\ln \sqrt[k]{e} =$

4. [Maximum mark: 6] ***[try to solve without GDC]***

Solve the following equations

(a) $\log_3(x+1) = 2$ [2]

(b) $\log(x+1) = 2$ [2]

(c) $\ln(x+1) = 2$ [2]

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5. [Maximum mark: 12] ***[try to solve without GDC]***

Solve the following equations

(a) (i) $\log_7(x+5)=0$ (ii) $\log_7(x+5)=1$ [4]

(b) (i) $\log(x+5)=0$ (ii) $\log(x+5)=1$ [4]

(c) (i) $\ln(x+5)=0$ (ii) $\ln(x+5)=1$ [4]

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6. [Maximum mark: 8]

Find the following integers

$\ln e^5$ ..... $\log 10^5$ .....

$e^{\ln 5}$ ..... $10^{\log 5}$ .....

$\log_3 3^5$ ..... $\log_a a^5$ .....

$3^{\log_3 5}$ ..... $a^{\log_a 5}$ .....

PROPERTIES OF LOGARITHMS

7. [Maximum mark: 12] ***[try to solve without GDC]***

Confirm the following properties for $x = 1000$ and $y = 100$

$\log xy = \log x + \log y$	LHS =
	RHS =
$\log \frac{x}{y} = \log x - \log y$	LHS =
	RHS =
$\log x^2 = 2\log x$	LHS =
	RHS =

8. [Maximum mark: 12]

Use your GDC to confirm the following properties for $x = 6$ and $y = 2$

$\ln xy = \ln x + \ln y$	LHS =
	RHS =
$\ln \frac{x}{y} = \ln x - \ln y$	LHS =
	RHS =
$\ln x^2 = 2\ln x$	LHS =
	RHS =

9. [Maximum mark: 28]

Let $\log x = a$, $\log y = b$ and $\log z = c$. Express the following in terms of a, b, c.

$\log xy$	
$\log \frac{x}{y}$	
$\log x^3$	
$\log xyz$	
$\log x^2 y$	
$\log \sqrt{x}$	
$\log \frac{xy}{z}$	
$\log 10x$	
$\log 100x$	
$\log \frac{y}{10}$	
$\log \frac{y}{100}$	
$\log \frac{xy}{10z}$	
$\log \frac{1}{z}$	
$\log \frac{x^2 y^7}{\sqrt{z}}$	

10. [Maximum mark: 28]

Let $\ln x = a$, $\ln y = b$ and $\ln z = c$. Express the following in terms of a, b, c.

$\ln xy$	
$\ln \frac{x}{y}$	
$\ln x^3$	
$\ln xyz$	
$\ln x^2 y$	
$\ln \sqrt{x}$	
$\ln \frac{xy}{z}$	
$\ln ex$	
$\ln e^2 x$	
$\ln \frac{y}{e}$	
$\ln \frac{y}{e^2}$	
$\ln \frac{xy}{ez}$	
$\ln \frac{1}{z}$	
$\ln \frac{x^2 y^7}{\sqrt{z}}$	

11. [Maximum mark: 6]

Let $a = \log x$, $b = \log y$, and $c = \log z$. Write log $\left(\frac{x^2\sqrt{y}}{z^3}\right)$ in terms of a, b and c.

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12. [Maximum mark: 6]

Let $p = \log x$, $q = \log y$ and $r = \log z$. Write $\log\left(\frac{x}{y^2\sqrt{z}}\right)$ in terms of p, q and r.

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13. [Maximum mark: 6]

Let $\ln a = p$, $\ln b = q$. Write the following expressions in terms of p and q.

(a) $\ln a^3 b$ (b) $\ln\left(\frac{\sqrt{a}}{b}\right)$ [3+3]

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14. [Maximum mark: 6]

Given that $p = \log_a 5$, $q = \log_a 2$, express the following in terms of p and/or q.

(a) $\log_a 10$ (b) $\log_a 8$ (c) $\log_a 2.5$ [2+2+2]

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15. [Maximum mark: 6]

(a) Let $\log_c 3 = p$ and $\log_c 5 = q$. Find an expression in terms of p and q for

(i) $\log_c 15$; (ii) $\log_c 25$. [4]

(b) Find the value of d if $\log_d 6 = \frac{1}{2}$. [2]

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16. [Maximum mark: 6]

Given that $\log_5 x = y$, express each of the following in terms of y.

(a) $\log_5 x^2$ (b) $\log_5\left(\frac{1}{x}\right)$ (c) $\log_5 \sqrt{x}$ [2+2+2]

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EXPONENTIAL EQUATIONS

17. [Maximum mark: 7]

(a) Solve the following exponential equations without using logs

(i) $3^{2x} = 9$ (ii) $10^{2x} = 100$ [2]

(b) Solve the equation $3^{2x} = 5$

(i) by using logarithm to the base 3

(ii) by using logarithm to the base 10, i.e. $\log$

(iii) by using logarithm to the base e, i.e $\ln$ [3]

(c) Solve the equation $10^{2x} = 5$, by using $\log$ [1]

(d) Solve the equation $e^{2x} = 5$, by using $\ln$ [1]

(Solve the equations above by using GDC to confirm the results)

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18. [Maximum mark: 8] ***[try to solve without GDC]***

Solve the following equations

(a) $2^{x+3} = 5$ [2]

(b) $10^{x+3} = 5$ [2]

(c) $e^{x+3} = 5$ [2]

(d) $2e^{x+3} = 5$ [2]

LOGARITHMIC EQUATIONS

19. [Maximum mark: 12] ***[try to solve without GDC]***

Solve the equations

(a) $\log x + \log(x+1) = \log 6$ [4]

(b) $\log x + \log(x+3) = 1$ [4]

(c) $\log(x+18) - \log x = 1$ [4]

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20. [Maximum mark: 12] ***[try to solve without GDC]***

Solve the equations

(a) $\log_2 x + \log_2(x+1) = \log_2 6$ [4]

(b) $\log_2 x + \log_2(x+1) = 1$ [4]

(c) $\log_2(x+5) - \log_2 x = 1$ [4]

21. [Maximum mark: [15] ***[try to solve without GDC]***

Solve the equations

(a) $\log(10x+20) - 2\log x = 1$ [5]

(b) $\log_2(x+14) - 2\log_2 x = 2$ [5]

(c) $\log_2(4x) + 2\log_2 x = 5$ [5]

22. [Maximum mark: 6] ***[try to solve without GDC]***

Solve $\log_2 x + \log_2(x-2) = 3$, for $x > 2$

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23. [Maximum mark: 6] ***[try to solve without GDC]***

Find the **exact** value of x in each of the following equations.

(a) $\log_a (3x + 5) = 2$ (b) $\ln (x + 2) = 3$. [3+3]

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B. Paper 2 questions (LONG)

24. [Maximum mark: 13] ***[try to solve without GDC]***

Solve the following equations.

(a) $\log_x 49 = 2$ [3]

(b) $\log_2 8 = x$ [2]

(c) $\log_{25} x = -\frac{1}{2}$ [3]

(d) $\log_2 x + \log_2(x - 7) = 3$ [5]

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25. [Maximum mark: 12]

Consider the function $f(x) = 10e^{0.3x} + 5$

(a) Write down the domain of f. [1]

(b) Find the y-intercept of the graph. [1]

(c) Find $f(5)$ correct to 3sf. [2]

(d) Find $f^{-1}(100)$ correct to 3sf. [2]

(e) Find the first integer value of x for which the value of y will exceed 120. [2]

(f) Find the value of $f(-20)$ and deduce the equation of the horizontal asymptote of the graph. [2]

(g) Write down the range of f. [2]

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26. [Maximum mark: 9]

The functions $f(x)$ and $g(x)$ are defined by $f(x) = e^x$ and $g(x) = \ln(1+2x)$.

(a) Write down $f^{-1}(x)$. [1]

(b) Find $g^{-1}(x)$. [3]

(c) Find $(f \circ g)(x)$. [2]

(d) Find $(f \circ g)^{-1}(x)$. [3]

27. [Maximum mark: 20]

The first 4 terms of a geometric sequence $\{u_n\}$ are

$$5 \qquad 15 \qquad 45 \qquad a$$

(a) Find the value of a [1]

Consider the new sequence $\{v_n\}$

$$ln\,5 \qquad ln\,15 \qquad ln\,45 \qquad ln\,a$$

(b) Write down the values of the terns v_1, v_2, v_3, v_4 correct to 3 s.f. [3]

(c) Find the differences $v_2 - v_1$, $v_3 - v_2$, $v_4 - v_3$ using the values found in (b).
What do you deduce? [3]

(d) Repeat the process (b) to (c) for the new sequence $\{w_n\}$ by using $\log$, the logarithm to the base 10, instead of $\ln$. Do you obtain a similar result? [4]

Consider now a geometric sequence $\{u_n\}$ with first term a and common ratio 3.

(e) Write down the first three terms of the sequence in terms of a. [2]

Define a new sequence $\{v_n\}$ as above (by using $\ln$).

(f) Show, by using its first three terms that $\{v_n\}$ is an arithmetic sequence. State the common difference. [4]

The general term of a geometric sequence is given by $u_n = u_1 r^{n-1}$

(g) State a proposition which can be derived by the process above and use the nth term and the $(n+1)^{\text{th}}$ term to support your statement. [3]

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28. [Maximum mark: 10]

Let $f(x) = \log_3 \frac{x}{2} + \log_3 16 - \log_3 4$, for $x > 0$.

(a) Show that $f(x) = \log_3 2x$. [2]

(b) Find the value of $f(0.5)$ and of $f(4.5)$. [2]

(c) On graph paper, **sketch** the graph of f, for $-5 \le x \le 5$, $-5 \le y \le 5$, using a scale of 1 cm to 1 unit on each axis. Indicate the equation of the asymptote. [2]

(d) Write down the value of $f^{-1}(0)$. [1]

The point A lies on the graph of f. At A, $x = 4.5$.

(e) On your diagram, sketch the graph of f^{-1}, noting clearly the image of point A. [3]

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